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Simple Approximations Of
Tube Anode Characteristics

No sophisticated software necessary. Try this simple approach to determine anode tube characteristics.

Many designers have proposed analytical functions to approximate the anode characteristics of a tube. In general these analytical functions depend on a certain number of parameters which must be optimized, using a regression method, to best fit the anode characteristics of a given tube. Nevertheless, many audio amateurs do not have such optimization software in their toolbox. For these audio amateurs this article introduces a simple alternative strategy, based on the solver available in Excel software and gives a complete description of the procedure to be used, as well as an illustrative example of this procedure, with the triode type ECC81.

PROCEDURE

1. First you must collect the data of the anode characteristics of the particular triode under consideration. You can derive triode data collection from direct measurements, using a modern digital curve tracer or tube data handbook. Triode data is arranged in a table where horizontal lines represent predefined anode to cathode voltage: V_{ak} and vertical columns, predefined grid to cathode voltage: V_{gk}. At the crossing of a given column and line—that is to say, for a given V_{gk} and V_{ak}—the resulting anode current I_a is indicated in the table. In other words, the table defines a matrix $\left\| a_{i,j} \right\| = \left\| I_a\left(V_{gk_j}; V_{ak_i}\right) \right\|$ $\forall i = 0, 1, \cdots, N$, $\forall j = 0, 1, \cdots M$.

As an example, **Table 1** shows the data for the triode type ECC81, using the Philips tube data handbook.

2. The second step involves choosing the appropriate analytical function. For triodes, I strongly suggest the one proposed by Norman L.Koren[1]. It i s a simple formula with five parameters, flexible enough to describe the anode characteristics of any type of triode with rather good accuracy.

$$E_1 = \frac{V_{ak}}{k_p} Log\left[1 + Exp\left[k_p\left[\frac{1}{m} + \frac{V_{gk}}{\left(k_{vb} + V_{ak}^2\right)^{\frac{1}{2}}}\right]\right]\right]$$

If $E_1 > 0$, then $I_a = \dfrac{E_1^{E_x}}{k_{g1}}$

TABLE 1: Collected tube data (Philips tube data handbook)
Triode type ECC81

Vak/ Vgk	0,00E+00	-1,00E+00	-2,00E+00	-3,00E+00	-4,00E+00	-5,00E+00	-6,00E+00	-7,00E+00
0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00
20	1,40E-03	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00
40	2,97E-03	4,75E-04	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00
60	4,75E-03	1,13E-03	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00
80	6,41E-03	2,02E-03	2,37E-04	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00
100	8,31E-03	2,97E-03	6,88E-04	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00
120	1,02E-02	4,39E-03	1,33E-03	9,49E-05	0,00E+00	0,00E+00	0,00E+00	0,00E+00
140	1,23E-02	5,93E-03	2,25E-03	5,22E-04	0,00E+00	0,00E+00	0,00E+00	0,00E+00
160	1,44E-02	7,57E-03	3,20E-03	9,49E-04	4,75E-05	0,00E+00	0,00E+00	0,00E+00
180	1,66E-02	9,37E-03	4,51E-03	1,66E-03	2,85E-04	0,00E+00	0,00E+00	0,00E+00
200		1,13E-02	5,96E-03	2,49E-03	7,12E-04	1,90E-04	0,00E+00	0,00E+00
220		1,35E-02	7,47E-03	3,54E-03	1,23E-03	4,51E-04	0,00E+00	0,00E+00
240			9,02E-03	4,70E-03	2,02E-03	8,31E-04	2,37E-04	0,00E+00
260			1,08E-02	5,93E-03	2,85E-03	1,31E-03	4,75E-04	1,19E-04
280			1,27E-02	7,36E-03	3,92E-03	1,90E-03	8,31E-04	2,37E-04
300				9,02E-03	4,98E-03	2,61E-03	1,14E-03	3,80E-04
320				1,07E-02	6,31E-03	3,49E-03	1,66E-03	7,12E-04
340					7,59E-03	4,39E-03	2,25E-03	1,07E-03
360					9,02E-03	5,46E-03	2,97E-03	1,52E-03
380						6,64E-03	3,68E-03	2,06E-03
400						7,83E-03	4,63E-03	2,73E-03
420							5,62E-03	3,44E-03
440							6,76E-03	4,27E-03
460								5,10E-03

TABLE 2: Calculated tube data (Norman L. Koren analytical formula)
Triode type ECC81

Fitting parameters : t1 = 231,0, t2 = 81,5 t3 = 4280,0, t4 = 1,15, t5 = 155,0

Vak/ Vgk	0,00E+00	-1,00E+00	-2,00E+00	-3,00E+00	-4,00E+00	-5,00E+00	-6,00E+00	-7,00E+00
0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00
20	1,30E-03	1,56E-04	4,13E-06	8,50E-08	1,73E-09	3,53E-11	7,20E-13	1,47E-14
40	2,90E-03	4,80E-04	2,10E-05	6,67E-07	2,07E-08	6,42E-10	1,99E-11	6,18E-13
60	4,63E-03	1,08E-03	8,40E-05	4,39E-06	2,19E-07	1,09E-08	5,44E-10	2,71E-11
80	6,45E-03	1,98E-03	2,59E-04	2,16E-05	1,66E-06	1,26E-07	9,61E-09	7,31E-10
100	8,34E-03	3,15E-03	6,24E-04	7,81E-05	8,63E-06	9,33E-07	1,01E-07	1,08E-08
120	1,03E-02	4,54E-03	1,23E-03	2,17E-04	3,24E-05	4,66E-06	6,66E-07	9,50E-08
140	1,23E-02	6,09E-03	2,08E-03	4,87E-04	9,41E-05	1,71E-05	3,07E-06	5,49E-07
160	1,44E-02	7,78E-03	3,16E-03	9,30E-04	2,23E-04	4,95E-05	1,07E-05	2,30E-06
180	1,64E-02	9,56E-03	4,42E-03	1,57E-03	4,53E-04	1,19E-04	3,00E-05	7,52E-06
200		1,14E-02	5,85E-03	2,40E-03	8,11E-04	2,46E-04	7,13E-05	2,03E-05
220		1,34E-02	7,42E-03	3,41E-03	1,32E-03	4,55E-04	1,48E-04	4,72E-05
240			9,09E-03	4,60E-03	1,99E-03	7,67E-04	2,77E-04	9,71E-05
260			1,09E-02	5,92E-03	2,82E-03	1,20E-03	4,75E-04	1,81E-04
280			1,27E-02	7,38E-03	3,80E-03	1,76E-03	7,57E-04	3,12E-04
300				8,95E-03	4,93E-03	2,46E-03	1,14E-03	5,03E-04
320				1,06E-02	6,18E-03	3,29E-03	1,63E-03	7,67E-04
340					7,56E-03	4,25E-03	2,23E-03	1,11E-03
360					9,03E-03	5,34E-03	2,96E-03	1,55E-03
380						6,55E-03	3,80E-03	2,09E-03
400						7,86E-03	4,75E-03	2,73E-03
420							5,81E-03	3,47E-03
440							6,98E-03	4,32E-03
460								5,27E-03

Else, $I_a = 0$

Where: $t_1=k_p$, $t_2=\mu$, $t_3=k_{vb}$, $t_4=E_x$, $t_5=k_{g1}$ are the five parameters.
This function is only valid for $V_{gk} \le 0$. For $V_{gk} > 0$, you must model the grid current, which is not considered here.

With this analytical function, and assuming you already have a rough idea of the parameter values, you can set up the same chart as in **Table 1**, but with the calculated anode current values.

In other words, the table defines a matrix $\left\| b_{i,j} \right\| = \left\| I_a \left(V_{gk_j}, V_{ak_i} \right) \right\|$

$$\forall i = 0,1,\cdots,N\,,\ \forall j = 0,1,\cdots M\ .$$

As an example, **Table 2** shows the triode type ECC81 data, using the Norman L. Koren analytical function and estimated parameters.

3. This third step involves the setup of an error function. **Tables 1** and **2** for a given grid to cathode voltage V_{gk} and anode to cathode voltage V_{ak} allow you to build the following error function:

$$S = \sum_{i,j} \left[a_{i,j} - b_{i,j} \right]^2$$

$$\forall i = 0,1,\cdots,N\,,\ \forall j = 0,1,\cdots M$$

This error function is a Euclidian positive norm. Fitting parameters are generally determined by minimizing the error function using the classical regression method. The smaller the S, the better the optimization of the parameters.

4. Next you need to determine the parameters. To do this, use the solver available in the Excel software and open a worksheet devoted to the particular triode of interest (see **Fig. 1**, the Excel worksheet for the triode type ECC81).

The worksheet is arranged in such a way that **Tables 1** and **2** are placed close to each other in order to have a natural correspondence between equivalent cells. Fitting parameters are declared in a dedicated area of the worksheet as well as the error function S. The optimization of the fitting parameters is performed using the solver available in the toolbox. The cell to minimize is the one in which the error function is available. The variables which must be optimized are the cells in which the fitting parameters are defined.

5. The final step involves the display of the anode characteristics. This is done on a graph attached to the worksheet. On this graph are superimposed the anode characteristics deduced from the tube data collection (blue graph) and from the calculated values according to the optimized fitting parameters (red graph). You can add additional information, such as the maximum anode power dissipation.

PENTODE APPLICATION

This procedure works very well with triodes, but you can also apply it with success to pentodes. The only difficulty is the choice of the appropriate analytical function. I suggest you use a combination of the analytical functions proposed by Norman L. Koren[1] and Menno van der Veen[2]. Koren's function models the cathode current because it behaves like the anode current of a triode. The van der Veen function models the current distribution function between the anode and the screen grid. The result is a simple formula with five fitting parameters, flexible enough to describe the anode and screen grid characteristics of any pentodes, with rather good accuracy.

$$E_1 = \frac{V_{g2k}}{k_p} Log\left[1 + Exp\left[k_p\left[\frac{1}{m} + \frac{V_{g1k}}{V_{g2k}}\right]\right]\right]$$

$$a = \left[\frac{2}{p} Arctg\frac{V_{ak}}{V_{g2k}}\right]^{\frac{1}{n}}, \text{with } 1 \le n \le 10$$

If $E_1>0$, then $I_a = \frac{E_1^{E_x}}{k_{g1}} a$

and $I_{g2} = \frac{E_1^{E_x}}{k_{g1}}[1-a]$

Else $I_a=0$ and $I_{g2}=0$

Where: $p1=k_p$, $p_2=\mu$, $p_3=E_x$, $p_4=k_{g1}$, $p_5=n$ are the five fitting parameters. This proposed analytical formula is only valid for $V_{g1k}\le 0$. For $V_{g1k}>0$, you must model the control grid current, which is not considered here.

IMPORTANT

Proposed analytical functions are pure designer's imagination and, unfortunately, do not rely on the principles. If these functions are convenient and accurate to determine tube working points, they are not as efficient when used to simulate tube dynamic behavior. This is particularly true when you try to quantify, by simulation, distortions. Distortions depend on partial derivatives of any order that cannot be deduced from these analytical functions because no care is explicitly taken to achieve this goal. This is a systematic mistake that designers should avoid because of its dangerous design consequences[3].

I hope some of you will try this method, and sincerely wish you good luck.

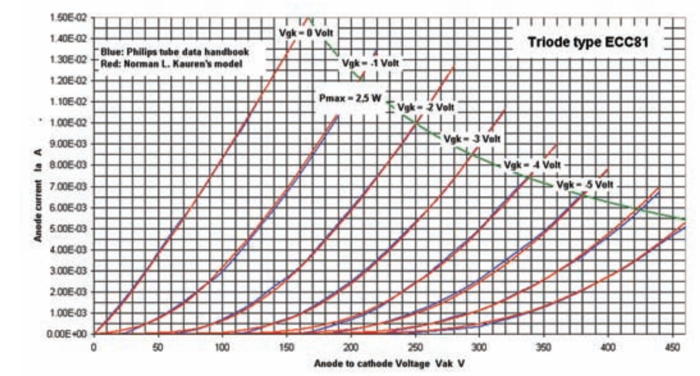

FIGURE 1: Excel worksheet to determine the anode characteristics of the triode type ECC81.

REFERENCES

1. Norman L. Koren, "Improved Vacuum Tube Models for SPICE Simulations" 5/96 *Glass Audio*.
2. Menno van der Veen, "Modelling power tubes and their interaction with output transformers," AES preprint 4643; 104th AES Convention 1998 Amsterdam.
3. Pierre Touzelet "Accurate non linear models of valve amplifiers including output transformers," AES preprint 6830; 120th Convention 2006 Paris.

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